



KEY

CANKAYA UNIVERSITY
Department of Mathematics

MATH 117 - Geometry For Architects

MIDTERM 1

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 70 minutes

Question	Grade	Out of
1		25
2		25
3		25
4		25
Total		100

IMPORTANT NOTES:

- 1) Please make sure that you have written your student number and name above.
- 2) Check that the exam paper contains 4 problems.
- 3) Show all your work. No points will be given to correct answers without reasonable work.

1) This question has two unrelated parts.

a) For the points $A(-6, 3)$, $B(3, -5)$, $C(-1, 5)$, verify that ABC is a right triangle and find its area.

$$|AB| = \sqrt{9^2 + 8^2} = \sqrt{145}$$

$$|AC| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$|BC| = \sqrt{4^2 + 10^2} = \sqrt{116}$$

since $|AB|^2 = |AC|^2 + |BC|^2$

$$145 = 29 + 116$$

it's a right triangle.

$$\text{Area} = \frac{\text{Base} \cdot \text{height}}{2} = \frac{\sqrt{29} \cdot \sqrt{116}}{2} = 29$$

b) Let $y = x^2 - 8x + 15$ be given. List the intercepts and test the equation for symmetry.

x-intercept : $x^2 - 8x + 15 = 0$
 $y=0$ $(x-3)(x-5)=0$ $x=3$
 $x=5$

y-intercept : $y=15$
 $x=0$

symm. wrt x :

$(x, -y)$ also satisfies
the eqn

symm. wrt y :

$(-x, y)$ satisfies the eqn

symm. wrt origin

$(-x, -y)$ satisfies the eqn.

$$-y = x^2 - 8x + 15 \neq y = x^2 - 8x + 15$$

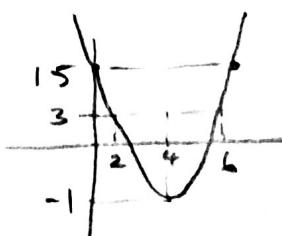
$$y = x^2 + 8x + 15 \neq y = x^2 - 8x + 15$$

$$-y = x^2 + 8x + 15 \neq y = x^2 - 8x + 15$$

It's not symm. wrt
x-axis, y-axis, origin

parabola.

x	y
0	15
2	3
4	-1
6	3
8	15



2) This question has two unrelated parts.

a) Find the center and the radius of the circle given by the following equation.

$$x^2 + y^2 + 6x - 4y - 3 = 0$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4$$
$$(x+3)^2 + (y-2)^2 = 16$$

Therefore center is $(-3, 2)$
radius is 4

b) Find the slope of the line passing through $(2, -1)$ and the midpoint of the points $(3, 5)$ and $(-7, -1)$.

Midpoint of $(3, 5)$ and $(-7, -1)$ is

$$M = \left(\frac{3-7}{2}, \frac{5-1}{2} \right) = (-2, 2)$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$= \frac{2 - (-1)}{-2 - 2}$$

$$= \frac{3}{-4}$$

3) This question has two unrelated parts.

a) Find an equation of the line passing through $(-1, -3)$ and parallel to the line $2y + 3x - 5 = 0$.

Rearrange $2y + 3x - 5 = 0$

$$y = -\frac{3x+5}{2} \quad m_1 = -\frac{3}{2}$$

Since they are parallel $m_1 = m_2 = -\frac{3}{2}$

$$y - y_0 = m(x - x_0)$$

$$y - (-3) = -\frac{3}{2}(x - (-1))$$

$$y + 3 = -\frac{3x}{2} - \frac{3}{2}$$

OR $y = -\frac{3x}{2} - \frac{9}{2}$

b) Find an equation of the line passing through $(5, 9)$ and perpendicular to the line $7y - 2x - 11 = 0$.

Rearrange $7y - 2x - 11 = 0$

$$y = \frac{2x+11}{7} \quad m_1 = \frac{2}{7}$$

Since they are perpendicular $m_1 \cdot m_2 = -1$

$$\frac{2}{7} \cdot m_2 = -1 \rightarrow m_2 = -\frac{7}{2}$$

$$y - y_0 = m(x - x_0)$$

$$y - 9 = -\frac{7}{2}(x - 5)$$

OR $y = -\frac{7}{2}x + \frac{53}{2}$

4) This question has two unrelated parts.

a) Write an equation of the line passing through the points $A(1, 2, -1)$ and $B(2, 3, 1)$.

Find a vector parallel to the line.

$$\begin{aligned}\vec{AB} &= (2-1)\mathbf{i} + (3-2)\mathbf{j} + (1-(-1))\mathbf{k} \\ &= \mathbf{i} + \mathbf{j} + 2\mathbf{k}\end{aligned}$$

Choose one of the pts.
and use:

$$x = x_0 + t^{\mathbf{v}_1}$$

$$y = y_0 + t^{\mathbf{v}_2}$$

$$z = z_0 + t^{\mathbf{v}_3}$$

$$-\infty < t < \infty$$

If we choose A:

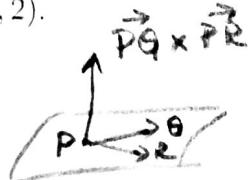
$$x = 1 + t$$

$$y = 2 + t$$

$$z = -1 + 2t$$

$$-\infty < t < \infty$$

b) Write an equation of the plane that contains the points $P(2, 0, 1)$, $Q(1, 1, 1)$ and $R(1, 0, 2)$.



$$\begin{aligned}\vec{PQ} &= (1-2)\mathbf{i} + (1-0)\mathbf{j} + (1-1)\mathbf{k} = -\mathbf{i} + \mathbf{j} \\ \vec{PR} &= (1-2)\mathbf{i} + (0-0)\mathbf{j} + (2-1)\mathbf{k} = -\mathbf{i} + \mathbf{k}\end{aligned}$$

$$\begin{aligned}\vec{PQ} \times \vec{PR} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = |1 \ 0 \ 0| \mathbf{i} - |-1 \ 0 \ 0| \mathbf{j} + |-1 \ 1 \ 0| \mathbf{k} \\ &= \mathbf{i} + \mathbf{j} + \mathbf{k}\end{aligned}$$

Choose one of the points and use:

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

Choosing $P(2, 0, 1)$

$$1(x-2) + (-1)(y-0) + 1(z-1) = 0$$

$$\text{OR } x + y + z = 3$$