



ÇANKAYA UNIVERSITY
Department of Mathematics

MATH 117 Geometry for Architects

2016-2017 Fall Semester

SECOND MIDTERM EXAMINATION
09.12.2016-10:00

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

DURATION: 100 minutes

KEY

Question	Grade	Out of
1		20
2		20
3		25
4		15
5		20
Total		100

IMPORTANT NOTES:

- 1) Please make sure that you have written your student number, name and signature above.
- 2) Check that the exam paper contains 5 problems.
- 3) Show all your work. No points will be given to correct answers without reasonable work.
- 4) Please TURN OFF your cellphones.
- 5) Calculators are NOT ALLOWED.
- 6) It is not allowed to leave the exam during the first 30 minutes.

- 1) a) (10 pts) Find a parametric equation for the line through the point $(1, 0, 2)$ and parallel to the vector $v = 2i + 3j - k$.

line through the point $(1, 0, 2)$ and in the direction of $\vec{v} = (2, 3, -1)$:

$$x = 1 + 2t$$

$$y = 0 + 3t$$

$$z = 2 - t$$

$$t \in \mathbb{R}$$

a parameter

$$\frac{x-1}{2} = \frac{y}{3} = \frac{z-2}{-1} = t$$

OR

a parameter

- b) (10 pts) Find a parametric equation for a line through the point $(2, -1, 0)$ and parallel to the plane $3x - y + 2z = 10$.

l : the required line.

If l is parallel to the plane, the direction vector of l and normal of the plane should be perpendicular.

\vec{n} := normal of the plane, $\vec{n} = (3, -1, 2)$

Let $\vec{v} = (a, b, c)$ be the direction vector of l .

$$\vec{v} \cdot \vec{n} = 0 \Rightarrow (a, b, c) \cdot (3, -1, 2) = 3a - b + 2c = 0$$

choose one of (a, b, c) satisfying this eqn.

Let $a = 1, b = -1, c = 1$. $\vec{v} = (1, -1, 1)$ is one possible vector.

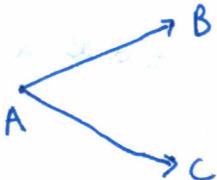
So the eqn of a line passing through $(2, -1, 0)$ and parallel to

\vec{v} is: $\frac{x-2}{1} = \frac{y-(-1)}{-1} = \frac{z-0}{1} = t$ t is a parameter.

$$\frac{x-2}{1} = \frac{y+1}{-1} = \frac{z}{1} = t$$

(Note that this is just only one of the lines satisfying the condition)

- 2) a) (10 pts) Find an equation for the plane through the points $A(1, -1, 2)$, $B(3, 0, 2)$, $C(-1, 2, 1)$.



$$\vec{AB} = (3-1, 0-(-1), 2-2) = (2, 1, 0)$$

$$\vec{AC} = (-1-1, 2-(-1), 1-2) = (-2, 3, -1)$$

needs a normal vector for the plane, say \vec{n} .
 $\vec{n} \perp \vec{AB}$ and $\vec{n} \perp \vec{AC} \Rightarrow \vec{n}$ is parallel to
 $\vec{AB} \times \vec{AC}$.

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ -2 & 3 & -1 \end{vmatrix} = i \begin{vmatrix} 0 & 2 \\ -2 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ -2 & 3 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} = -i - j(-2) + 8k$$

without loss of generality one can choose $\vec{n} = (-1, 2, 8)$

so the eqn for the plane is $[(x, y, z) - (1, -1, 2)] \cdot \vec{n} = 0$

$$(x-1) \cdot (-1) + (y+1) \cdot 2 + (z-2) \cdot 8 = 0$$

$$-x+1+2y+2+8z-16=0 \Rightarrow \boxed{-x+2y+8z=13}$$

- b) (10 pts) Find a plane through the point $P(-2, 1, -1)$ and with a normal which is parallel to the line of intersection of the planes $x - 3y + z = 2$ and $3x + y - 2z = 4$.

$$P_1: x - 3y + z = 2 \quad \vec{n}_1 := \text{normal of } P_1 = (1, -3, 1)$$

$$P_2: 3x + y - 2z = 4 \quad \vec{n}_2 := \text{normal of } P_2 = (3, 1, -2)$$

direction vector of the line of intersection will be parallel to $\vec{n}_1 \times \vec{n}_2$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 3 & 1 & -2 \end{vmatrix} = i \begin{vmatrix} -3 & 1 \\ 1 & -2 \end{vmatrix} - j \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} + k \begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix}$$

$$= 5i + 5j + 10k.$$

If normal is parallel to the line of intersection, normal should be parallel to $\vec{n}_1 \times \vec{n}_2$. WLOG the required normal can be taken to be $\vec{n}_1 \times \vec{n}_2$. $\vec{n} = (5, 5, 10)$

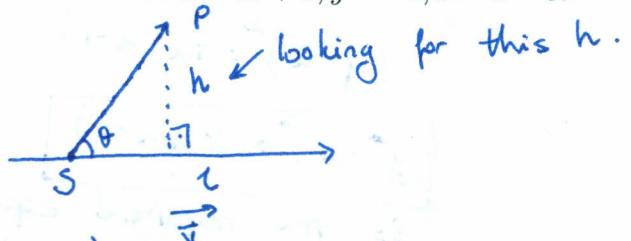
$$\text{The plane eqn. is } \vec{n}[(x, y, z) - (-2, 1, -1)] = 0$$

$$5(x+2) + 5(y-1) + 10(z+1) = 0$$

$$x+2+y-1+2z+2 = 0$$

$$\boxed{x+y+2z+3=0}$$

- 3) a) (10 pts) Find the distance from the point $P(1, -1, 3)$ to the line
 $l : x = 2t + 1, y = -t, z = 2 - 3t$



$$h = |\vec{SP}| \cdot \sin \theta$$

$$h = \frac{|\vec{SP} \times \vec{v}|}{|\vec{v}|}$$

$$S = (1, 0, 2)$$

$$\vec{SP} = (1-1, -1-0, 3-2) \\ = (0, -1, 1)$$

$$\vec{v} = (2, -1, -3)$$

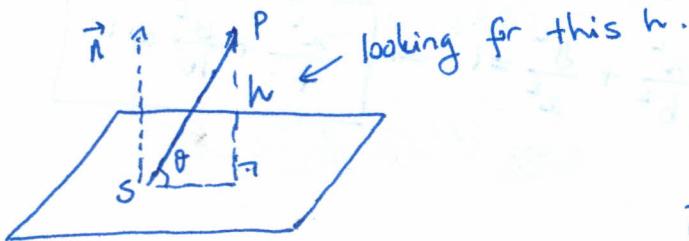
$$|\vec{v}| = \sqrt{4+1+9} \\ = \sqrt{14}$$

$$\vec{SP} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & -1 & 1 \\ 2 & -1 & -3 \end{vmatrix} = i \begin{vmatrix} -1 & 1 \\ -1 & -3 \end{vmatrix} - j \begin{vmatrix} 0 & 1 \\ 2 & -3 \end{vmatrix} + k \begin{vmatrix} 0 & 1 \\ -1 & -1 \end{vmatrix} \\ = 4i + 2j + 2k$$

$$|\vec{SP} \times \vec{v}| = \sqrt{16+4+4} = \sqrt{24}$$

$$\text{so } h = \frac{\sqrt{24}}{\sqrt{14}} = \sqrt{\frac{12}{7}}$$

- b) (10 pts) Find the distance from the point $P(0, -1, -5)$ to the plane $x - 3y + z = 10$.



$$h = |\vec{SP}| \cdot \sin \theta = \frac{|\vec{SP} \cdot \vec{n}|}{|\vec{n}|}$$

a point S on the plane $(1, -3, 0)$

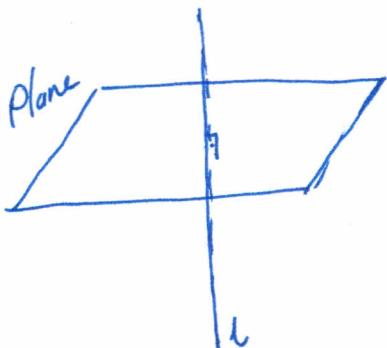
$$\vec{SP} = (0-1, -1+3, -5-0) \\ = (-1, 2, -5)$$

$$\vec{n} = (1, -3, 1) \quad \vec{SP} \cdot \vec{n} = -1 \cdot 1 + 2 \cdot (-3) + (-5) \cdot 1 \\ = -12$$

$$|\vec{n}| = \sqrt{1+9+1} \\ = \sqrt{11}$$

$$h = \frac{|\vec{SP} \cdot \vec{n}|}{|\vec{n}|} = \frac{12}{\sqrt{11}}$$

- c) (5 pts) Is the line $x = -2t, y = 1 - 5t, z = 3 + 2t$ perpendicular to the plane $2x + 3y - 5z = 21$? Give reasons for your answer.

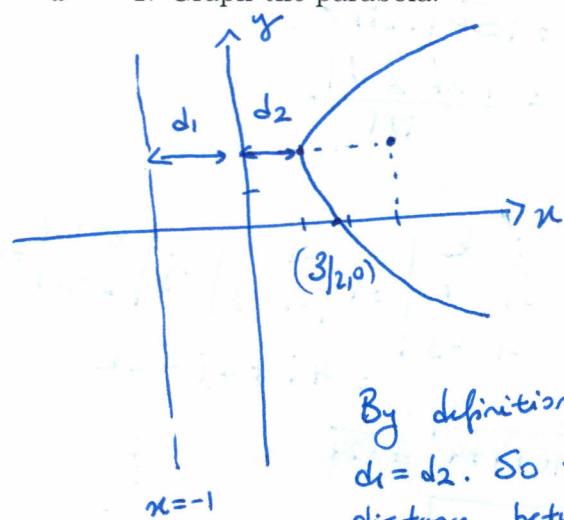


if the line is perpendicular to the plane
 then direction vector of the line
 should be parallel to the normal of the plane.

Direction vector of the line is $(-2, -5, 2) = \vec{v}$
 Normal of the plane is $(2, 3, -5) = \vec{n}$

$\vec{n} \neq c \cdot \vec{v}$ for some $c \in \mathbb{R}$, so they are not parallel.
 One can also consider the angle between them

- 4) a) (5 pts) Find the equation of the parabola with focus at $(3, 2)$ and directrix the line $x = -1$. Graph the parabola.



By definition

$d_1 = d_2$. So vertex is $(1, 2)$

distance between foci and vertex is $a = 2$

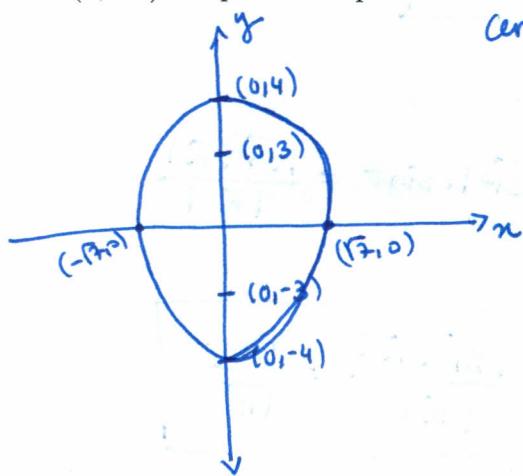
$$(y-2)^2 = 4a(x-1)$$

$$(y-2)^2 = 8(x-1)$$

is the required equation.

(If $y=0$ $x = \frac{3}{2}$
parabola passes through
the point $(3, 2)$)

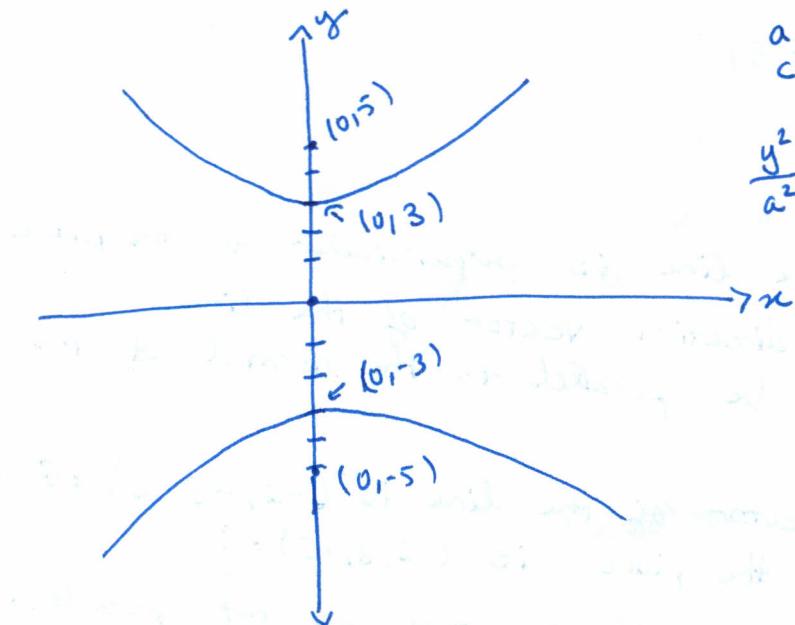
- b) (5 pts) Find the equation of the ellipse having one focus at $(0, 3)$ and one vertex at $(0, -4)$. Graph the ellipse.



Center should be at $(0, 0)$
 $a = 4$ $b = 3$ $c^2 = a^2 - b^2 \Rightarrow b^2 = 16 - 9 = 7$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \Rightarrow \boxed{\frac{x^2}{7} + \frac{y^2}{16} = 1}$$

- c) (5 pts) Find the equation of the hyperbola with center at origin, one focus at $(0, 5)$ and one vertex at $(0, -3)$. Graph the hyperbola.



$$a = 3 \quad c = 5 \quad b^2 = c^2 - a^2 = 25 - 9 = 16$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \boxed{\frac{y^2}{9} - \frac{x^2}{16} = 1}$$

5) Classify the given equations: Do they define a circle, a parabola, an ellipse, a hyperbola or neither of them ? Give reasons for your answer.

a) (6 pts) $4y^2 - 12x - 8y + 16 = 0$

$$4(y^2 - 2y + 1 - 1) - 12x + 16 = 0$$

$$4(y-1)^2 - 12x + 12 = 0$$

$$4(y-1)^2 = 12(x-1)$$

This is an eqn of a parabola.

b) (7 pts) $5x^2 - 30x + 9y^2 + 36y + 36 = 0$

$$5(x^2 - 6x + 9 - 9) + 9(y^2 + 4y + 4) = 0$$

$$5(x-3)^2 + 9(y+2)^2 = 45$$

$$\frac{(x-3)^2}{9} + \frac{(y+2)^2}{5} = 1 \quad \text{This is an eqn of an ellipse.}$$

c) (7 pts) $2x^2 - 4x + 2 - 3y^2 - 6y - 9 = 0$

$$2(x^2 - 2x + 1) - 3(y^2 + 2y + 1) - 6 = 0$$

$$2(x-1)^2 - 3(y+1)^2 = 6$$

$$\frac{(x-1)^2}{3} - \frac{(y+1)^2}{2} = 1 \quad \text{This is an eqn. of a hyperbola.}$$