ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 117 Geometry for Architects

2016-2017 Fall Semester

FIRST MIDTERM EXAMINATION

4.11.2016

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE: KEY

DURATION: 100 minutes

Question	Grade	Out of
1		15
2		20
3		25
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number, name and signature above.

2) Check that the exam paper contains 5 problems.

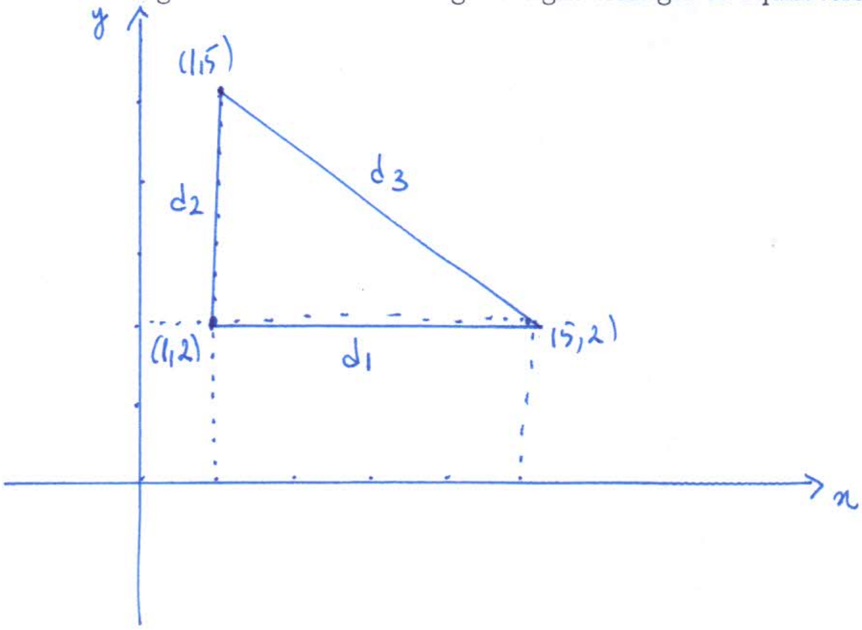
3) Show all your work. No points will be given to correct answers without reasonable work.

4) Please TURN OFF your cellphones.

5) Calculators are NOT ALLOWED.

6) It is not allowed to leave the exam during the first 30 minutes.

1) a)(10 pts) Consider the three points $A = (1, 2)$, $B = (5, 2)$ and $C = (1, 5)$. Form the triangle ABC. Is this triangle a right triangle or equilateral triangle? Explain.



$$\left.\begin{array}{l} d_1 = \sqrt{(5-1)^2 + (2-2)^2} = 4 \\ d_2 = \sqrt{(1-1)^2 + (5-2)^2} = 3 \\ d_3 = \sqrt{(5-1)^2 + (2-5)^2} = 5 \end{array}\right\}$$ as $d_1^2 + d_2^2 = d_3^2$

It's a right triangle.

b)(5 pts) Find the distance between the point $(2, 3)$ and the midpoint of the line segment from $(1, 5)$ and $(3, 1)$.

midpoint of the line segment from (1,5) and (3,1)

is $M = \left(\frac{3+1}{2}, \frac{1+5}{2}\right) = (2, 3)$

Distance between (2,3) and M is zero as they are same point.

(OR $d = \sqrt{(2-2)^2 + (3-3)^2} = 0$)

2) a) (10 pts) Test $y = \frac{x^3}{x^2+1}$ for symmetry: Is the graph of y symmetric with respect to y-axis, x-axis or origin?

Symmetry w.r.t x-axis: whenever (x,y) on the graph, $(x,-y)$ should be on the graph. $-y = \frac{x^3}{x^2+1}$ do not have the same graph as $y = \frac{x^3}{x^2+1}$

Symmetry w.r.t y-axis: whenever (x,y) on the graph, $(-x,y)$ should be on the graph. $y = \frac{(-x)^3}{x^2+1} = \frac{-x^3}{x^2+1}$, graphs are different.

symmetry w.r.t origin: whenever (x,y) on the graph $(-x,-y)$ should be on the graph. $-y = \frac{(-x)^3}{x^2+1} = \frac{-x^3}{x^2+1} \Rightarrow y = \frac{x^3}{x^2+1}$

So $(-x,-y)$ is on the graph.

graph of $y = \frac{x^3}{x^2+1}$ is symmetric w.r.t origin.

b) (10 pts) Find the intercepts and then graph the equation $y = x^2 + 3x$.

Intercepts: $x=0 \quad y=0 \quad (0,0)$

$y=0 \quad x^2+3x = x(x+3) = 0 \Rightarrow x=0,\ x=-3$

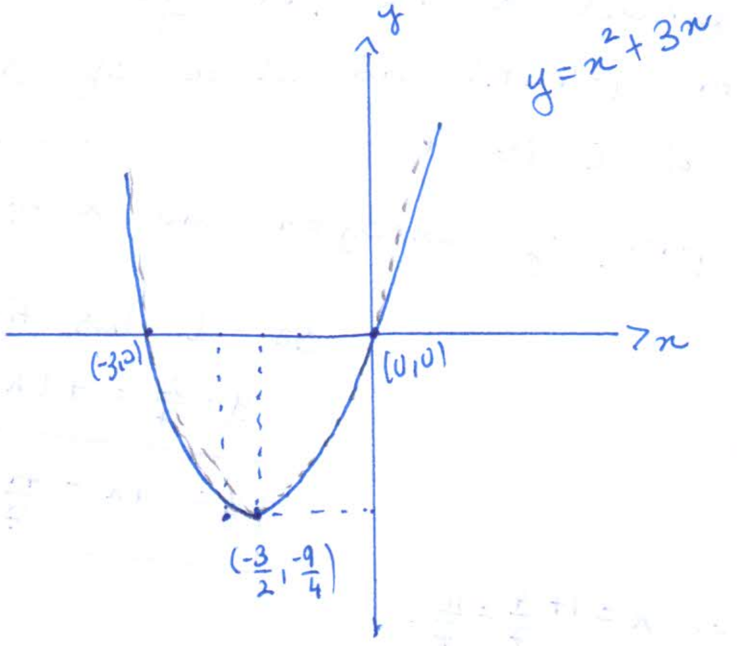
So x-interepts are $(0,0)$, $(-3,0)$ and y-intercept is $(0,0)$.

$y = x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2$

$y = \left(x+\frac{3}{2}\right)^2 - \frac{9}{4}$

so vertex is

$\left(-\frac{3}{2}, -\frac{9}{4}\right)$



3) a) (5 pts) Find an equation of the line passing through the points $(-2,3)$ and $(2,4)$.

slope of the line: $m = \frac{4-3}{2-(-2)} = \frac{1}{4}$

Eqn of the line: $y-4 = \frac{1}{4}(x-2)$

$$\boxed{y = \frac{x}{4} - \frac{1}{2} + 4 = \frac{x}{4} + \frac{7}{2}}$$

b)(10 pts) Find an equation of a line passing through the point $(-1,4)$ and is perpendicular to the line $2x+5y=1$.

l: required line

If l is perpendicular to $2x+5y=1$, it has slope equal to $\frac{5}{2}$

So eqn of l: $y-4 = \frac{5}{2}(x-(-1))$

$$\boxed{y = \frac{5}{2}x + \frac{5}{2} + 4 = \frac{5}{2}x + \frac{13}{2}}$$

(slope of $2x+5y=1$: as $y = -\frac{2}{5}x + 1$, $m = -\frac{2}{5}$)

c) (10 pts) Find an equation of a line passing through the intersection point of the lines $2x+3y=4$ and $x-2y=1$ and is parallel to the line $y=4x+3$.

l: required line

If l is parallel to the line $y=4x+3$, l has the same slope with $y=4x+3$ and slope of $y=4x+3$ is 4.

So slope of l is 4

Intersection point of $2x+3y=4$ and $x-2y=1$

$$\begin{array}{rl} & 2x+3y=4 \\ -2/ & x-2y=1 \\ + & \\ \hline & 7y=2 \\ & y=\frac{2}{7} \end{array}$$

$x - 2\cdot\frac{2}{7} = 1 \Rightarrow x = 1 + \frac{4}{7} = \frac{11}{7}$

So l has the eqn

$y - \frac{2}{7} = 4\left(x - \frac{11}{7}\right)$

$$\boxed{y = 4x - \frac{44}{7} + \frac{2}{7} = 4x - 6}$$

4) a) (10 pts) Find the radius and center of the circle

$$x^2 + y^2 - 3x + 2y = 1$$

$$\underbrace{x^2 - 3x + \left(-\frac{3}{2}\right)^2} - \frac{9}{4} + \underbrace{y^2 + 2y + 1} - 1 = 1$$

$$\left(x - \frac{3}{2}\right)^2 + (y+1)^2 = 1 + 1 + \frac{9}{4} = \frac{17}{4}$$ is the standard eqn for circle.

so center is $\left(\frac{3}{2}, -1\right)$ and radius is $\frac{\sqrt{17}}{2}$

b) (10 pts) The end points of the diameter of a circle are $(-2, 3)$ and $(1, 2)$. Find the center and radius of the circle. Write the standart equation of that circle.

Center is the midpoint of the diameter, so it's $\left(\frac{-2+1}{2}, \frac{3+2}{2}\right)$
$= \left(-\frac{1}{2}, \frac{5}{2}\right)$

radius is the half of the diameter, so it is $\frac{\sqrt{(-2-1)^2 + (3-2)^2}}{2} = \frac{\sqrt{10}}{2}$

standart equation of the circle is $\left(x - \left(-\frac{1}{2}\right)\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{10}{4} = \frac{5}{2}$

5) a)(10 pts) If the vectors $v_1 = (1, -2, a)$ and $v_2 = (3, 2, -1)$ are perpendicular, what is a?

If $V_1 \perp V_2$ then dot product $V_1 \cdot V_2 = 0$

so $V_1 \cdot V_2 = (1, -2, a)(3, 2, -1) = 0$

$= 1.3 - 2.2 + a.(-1) = 0$

$\Rightarrow \quad a = -1$

b) (10 pts) Calculate the cross product of the vectors $v_1 = (-1, 2, -2)$ and $v_2 = (1, 0, 2)$.

$$V_1 \times V_2 = \begin{vmatrix} i & j & k \\ -1 & 2 & -2 \\ 1 & 0 & 2 \end{vmatrix} = i \begin{vmatrix} 2 & -2 \\ 0 & 2 \end{vmatrix} - j \begin{vmatrix} -1 & -2 \\ 1 & 2 \end{vmatrix} + k \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix}$$

$$= 4i - j(-2+2) + k(-2)$$

$$= 4i - 2k$$

$V_1 \times V_2 = (4, 0, -2)$